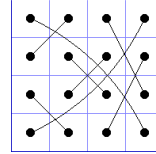


Theorem 11 [connection figure (1234 2567 8653 4871)]



(i) Let  $k, r, s$  be natural numbers with  $s < r$  and

- (I)  $2s < r$  and either
- (AI)  $1 < k < s+1$  (BI)  $s+1 < k < 2s+1$
- (CI)  $2s+1 < k < r+1$  (DI)  $r+1 < k < r+2s+1$
- (EI)  $r+2s+1 < k < 2r+1$  (FI)  $2r+1 < k < 2r+s+1$
- (GI)  $2r+s+1 < k < 2r+2s+1$  (HI)  $2r+2s+1 < k$ ,

or

- (II)  $r < 2s$  and either
- (AII)  $1 < k < s+1$  (BII)  $s+1 < k < r+1$  (CII)  $r+1 < k < 2s+1$  (DII)  $2s+1 < k < 2r+1$
- (EII)  $2r+1 < k < r+2s+1$  (FII)  $r+2s+1 < k < 2r+s+1$  (GII)  $2r+s+1 < k < 2r+2s+1$  (HII)  $2r+2s+1 < k$ .

Then the 16 numbers

$$(*) \quad 1, 1+s, 1+2s, 1+r, 1+r+2s, 1+2r, 1+2r+s, 1+2r+2s, \\ k, k+s, k+2s, k+r, k+r+2s, k+2r, k+2r+s, k+2r+2s$$

are pairwise different and (\*) is a symmetric subset of  $\{1, \dots, N\}$ ,  $N = k+2r+2s$ . Moreover, there are 16 different general 4x4 magic squares  $M01, M02, \dots, M16$  with entries from (\*) and connection figure (1234 2567 8653 4871), namely

$$M01 = \begin{matrix} 1+s & 1+2r & k+2r+2s & k+s \\ k+2s & k+r & 1+r & 1+2r+2s \\ k+2r & k+r+2s & 1+r+2s & 1 \\ 1+2r+s & 1+2s & k & k+2r+s \end{matrix}$$

$$M02 = \begin{matrix} 1+s & 1+2r+2s & k+2r & k+s \\ k & k+r+2s & 1+r+2s & 1+2r \\ k+2r+2s & k+r & 1+r & 1+2s \\ 1+2r+s & 1 & k+2s & k+2r+s \end{matrix} \quad \text{(in } M01 \text{ the [virtual] summand } 0s \text{ has been exchanged with } 2s)$$

$M03$  and  $M04$  are derived from  $M01$  and  $M02$  by exchanging 1 and  $k$ ,  $M05, M06, M07$ , and  $M08$  arise from  $M01, M02, M03$  and  $M04$  by exchange of  $r$  and  $s$ , and finally,  $M08, M09, \dots, M16$  are the mirror images of  $M01, M02, \dots, M08$  by horizontal reflection.

(ii) Every general 4x4 magic square  $M$  with entries from a symmetric subset of  $\{1, \dots, N\}$ , with connection figure (1234 2567 8653 4871), and with entry 1, is of the form either  $M1, M2, \dots, M15$ , or  $M16$ ; and the corresponding parameters  $k, r, s$  for this subset fulfil the inequalities either AI, BI, CI, DI, EI, FI, GI, HI, AII, BII, CII, DII, EII, FII, GII, or HII.

Proof

(i) is verified easily; (ii) follows, when the linear equations for  $M$  are solved.

Remark 1

For each symmetric subset of  $\{1, \dots, N\}$  containing the number 1, and allowing a general 4x4 magic square of connection figure (1234 2567 8653 4871) there are exactly 16 different general 4x4 magic squares with entries from this subset. The smallest  $N$ , with a symmetric subset allowing a general 4x4 magic square with connection figure (1234 2567 8653 4871) and 1 as an entry, is  $N=18$ . From  $N=18$  to  $N=29$  every case AI, ..., HII occurs.

Remark 2

(i) With  $s=3r$  in (\*) one has the 16 numbers of (\*) from Theorem 06. The map

$$\begin{matrix} c01 & c02 & c03 & c04 & & c06 & c02 & c10 & c14 \\ c05 & c06 & c07 & c08 & & c01 & c05 & c13 & c09 \\ c09 & c10 & c11 & c12 & \rightarrow & c04 & c08 & c16 & c12 \\ c13 & c14 & c15 & c16 & & c07 & c03 & c11 & c15 \end{matrix}$$

brings any general magic 4x4 square of connection figure (1122 3456 7878 6543) onto a general 4x4 magic square of connection figure (1234 2567 8653 4871).

(ii) With  $s=4r$  in (\*) one has the 16 numbers of (\*) from Theorem 07. The map

```

c01 c02 c03 c04      c06 c13 c04 c14
c05 c06 c07 c08      c11 c08 c12 c02
c09 c10 c11 c12  --> c10 c05 c09 c03
c13 c14 c15 c16      c07 c16 c01 c15

```

brings any general magic 4x4 square of connection figure (1122 3456 6783 8547) onto a general 4x4 magic square of connection figure (1234 2567 8653 4871)

(iii) There is an imbedding map  $i$  of the set of GMS of connection figure (1234 2567 8653 4871) into the set of GMS of connection figure (1122 3443 5665 7788), defined by:

```

          c01 c02 c03 c04      c12 c03 c14 c09
          c05 c06 c07 c08      c06 c13 c04 c11
i:  c09 c10 c11 c12  -->  c10 c01 c16 c07.
          c13 c14 c15 c16      c08 c15 c02 c05

```

This follows from seven additional equations, valid for every general magic square of connection figure (1234 2567 8653 4871):

$c01+c03+c10+c12 = 2(N+1)$ ,  $c01+c04+c08+c09 = 2(N+1)$ ,  $c04+c06+c11+c13 = 2(N+1)$ ,  
 $c02+c04+c14+c16 = 2(N+1)$ ,  $c05+c07+c09+c11 = 2(N+1)$ ,  $c05+c12+c13+c14 = 2(N+1)$ ,  
 $c06+c08+c10+c12 = 2(N+1)$ .