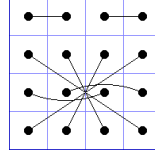


Theorem 6 [connection figure (1122 3456 7878 6543)]



(i) Let k, r be natural numbers with either

- (A) $1 < k < r+1$, (B) $r+1 < k < 2r+1$,
 (C) $2r+1 < k < 3r+1$, (D) $3r+1 < k < 5r+1$
 (E) $5r+1 < k < 6r+1$, (F) $6r+1 < k < 7r+1$,
 (G) $7r+1 < k < 8r+1$, (H) $8r+1 < k$.

Then the 16 numbers

- (*) $1, r+1, 2r+1, 3r+1, 5r+1, 6r+1, 7r+1, 8r+1,$
 $k, k+r, k+2r, k+3r, k+5r, k+6r, k+7r, k+8r$

are pairwise different and (*) is a symmetric subset of $\{1, \dots, N\}$, $N=k+8r$, with difference vector, casewise:

- (A) $k-1, r-k+1, k-1, r-k+1, k-1, r-k+1, k-1, 2r-k+1, k-1, r-k+1, k-1, r-k+1, k-1, r-k+1, k-1$
 (B) $r, k-r-1, 2r-k+1, k-r-1, 2r-k+1, k-r-1, r, 2r-k+1, r, k-r-1, 2r-k+1, k-r-1, 2r-k+1, k-r-1, r$
 (C) $r, r, k-2r-1, 3r-k+1, k-2r-1, r, 3r-k+1, k-2r-1, 3r-k+1, r, k-2r-1, 3r-k+1, k-2r-1, r, r$
 (D) $r, r, r, k-3r-1, r, 4r-k+1, k-3r-1, 4r-k+1, k-3r-1, 4r-k+1, r, k-3r-1, r, r, r$
 (E) $r, r, r, 2r, k-5r-1, 6r-k+1, k-5r-1, 6r-k+1, k-5r-1, 6r-k+1, k-5r-1, 2r, r, r, r$
 (F) $r, r, r, 2r, r, k-6r-1, 7r-k+1, k-6r-1, 7r-k+1, k-6r-1, r, 2r, r, r, r$
 (G) $r, r, r, 2r, r, r, k-7r-1, 8r-k+1, k-7r-1, r, r, 2r, r, r, r$
 (H) $r, r, r, 2r, r, r, r, k-8r-1, r, r, r, 2r, r, r, r$.

Moreover, there are 4 different general 4×4 magic squares M_1, M_2, M_3, M_4 with entries from (*) and connection figure (1122 3456 7878 6543), namely

$$M_1 = \begin{matrix} & k+6r & 2r+1 & 6r+1 & k+2r \\ & k+r & 3r+1 & 5r+1 & k+7r \\ & 8r+1 & k+8r & k & 1 \\ & r+1 & k+3r & k+5r & 7r+1, \end{matrix}$$

$$M_2 = \begin{matrix} & 6r+1 & k+2r & k+6r & 2r+1 \\ & r+1 & k+3r & k+5r & 7r+1 \\ & k+8r & 8r+1 & 1 & k \\ & k+r & 3r+1 & 5r+1 & k+7r. \end{matrix}$$

M_3 and M_4 , are obtained from M_1 and M_2 by reflection at a vertical axis.

(ii) Every general magic square M with entries from a symmetric subset $\{1, \dots, N\}$, with connection figure (1122 3456 7878 6543), and with entry 1, is of the form either M_1, M_2, M_3 , or M_4 , and the corresponding difference vector of this subset is of the form either A, B, C, D, E, F, G, or H.

Proof

- (i) can be verified by a simple calculation,
 (ii) can be proved by solving the linear equations for M .

Remark

For each symmetric subset of $\{1, \dots, N\}$ containing the number 1, and allowing a general 4×4 magic square of connection figure (1122 3456 7878 6543) there are exactly 4 different general 4×4 magic squares with entries from this subset. There exists an imbedding from the set of general 4×4 magic squares of connection figure (1122 3456 7878 6543) into the set of general 4×4 magic squares of connection figure (1234 2567 8653 4871) [see Theorem 11, Remark 2 (i)].